Hybrid Event Shaping to Stabilize Periodic Hybrid Orbits

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***Abstract*—Many controllers for legged robotic systems leverage open- or closed-loop control at discrete hybrid events to enhance stability. These controllers appear in several well studied phenomena such as the Raibert stepping controller, paddle juggling, and swing leg retraction. This work introduces hybrid event shaping (HES): a generalized method for analyzing and designing stable hybrid event controllers. HES utilizes the saltation matrix, which gives a closed-form equation for the effect that hybrid events have on stability. We also introduce shape parameters, which are higher order terms that can be tuned completely independently of the system dynamics to promote stability. Optimization methods are used to produce values of these parameters that optimize a stability measure. Hybrid event shaping captures previously developed control methods while also producing new optimally stable trajectories without the need for continuous-domain feedback.**

I. Introduction

In general, the walking and running gaits of legged robots are naturally unstable and challenging to control. Hybrid systems such as these are difficult to work with due to the discontinuities in state and dynamics that occur at hybrid events, such as toe touchdown, which violate assumptions of standard controllers designed for purely continuous systems.

Several works have examined the utility of controlling hybrid event conditions to improve system stability without any closed-loop continuous-domain control [2,3]. For example, [2] found that for the paddle juggler system, paddle acceleration at impact uniquely determines the local stability properties of a periodic trajectory, Fig. 1. So far, however, these results have only been produced for the specific problem structure and are not generalizable to more complex systems.

In this work, we propose the concept of hybrid event shaping (HES), which describes how hybrid event parameters can be chosen to affect the stability properties of a periodic orbit. We also propose methods to produce values of these hybrid event parameters to optimize a stability measure of a trajectory.

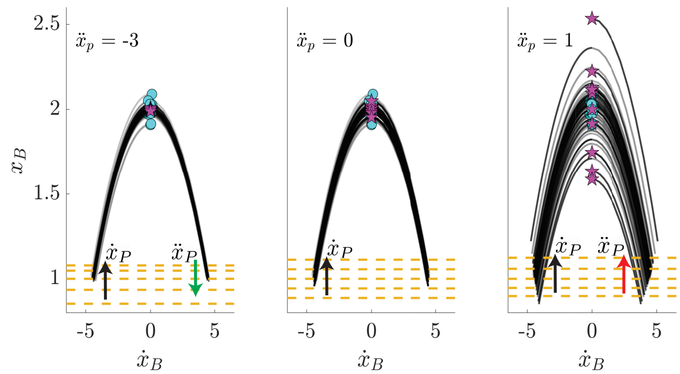

Fig. 1. The paddle juggler system [2] has no control authority while the ball is in the air. The paddle acceleration at impact determines the convergence/divergence of the system from initial points (cyan dots) to the final states (magenta stars) after 5 cycles. This example underscores how hybrid event shaping can stabilize a periodic hybrid system.

II. Preliminaries

Following the adaptation of [4] in [1], we define a hybrid system as a tuple $\mathcal{H} := (\mathcal{J}, \mathit{\Gamma}, \mathcal{D}, \mathcal{F}, \mathcal{G}, \mathcal{R})$. The variational equations of hybrid events are characterized by the saltation matrix $\Xi_{(I,J)}(\bar{t}_i, x(\bar{t}_i), u(\bar{t}_i))$, which approximates the first-order change in perturbations in state before a hybrid event at $\delta x(\bar{t}_i)$ to perturbations afterward $\delta x(\underline{t}_{i+1})$ [5]. Following the formulation from [5], the saltation matrix is,

$$\Xi = D_x R + \frac{(F_J - D_x R \cdot F_I - D_t R) D_x g}{D_t g + D_x g F_I} \tag{1}$$

A dynamical system has a periodic trajectory (orbit) if it repeats itself after period T. Initial perturbations δx_0 can be mapped to perturbations δx_T after period T by a linearized mapping known as the monodromy matrix, Φ which can be computed by sequentially composing the linearized variational equations in each continuous domain (A) and the saltation matrices (Ξ) at each hybrid event [6]:

$$\Phi = \Xi_{(N,1)} A_N \ldots \Xi_{(2,3)} A_2 \Xi_{(1,2)} A_1 \tag{2}$$

The monodromy matrix determines local asymptotic orbital stability. For nonautonomous systems, stability is determined by the maximum magnitude of the eigenvalues, $\max(|\lambda|)$ [7]. We refer to this as the stability measure, ψ, where a trajectory is stable when $\psi < 1$. Autonomous systems always have an eigenvalue that is equal to 1 since for non-time varying dynamics, perturbations along the flow of the orbit will by definition map back to themselves after period T [7]. Assuming non-convergence in this direction is allowable, ψ for autonomous systems is based on the remaining eigenvalues.

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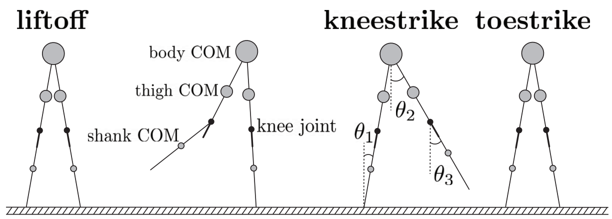

Fig. 2. Biped walker system with kneestrike and toestrike hybrid events.

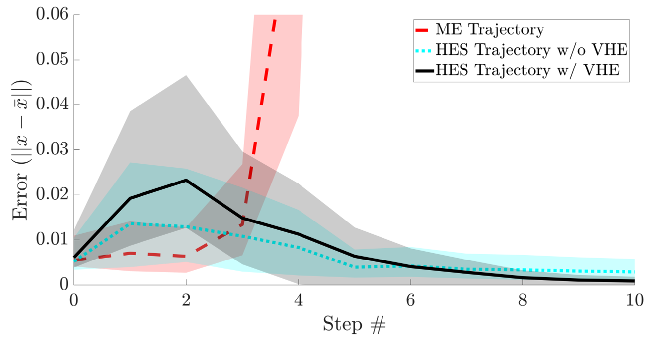

Fig. 3. Error of perturbed Minimum Energy (ME), Hybrid Event Shaping without virtual hybrid events (HES w/o VHE) and HES w/ VHE trajectories over several steps. Bold lines show average error at each step and shaded regions indicate ± 1 standard deviation. The initial increase in error of the HES trajectories is allowable and is not considered by the stability measure.

III. Methods

The saltation matrix allows for an explicit understanding of how to perform **hybrid event shaping** (HES), i.e. choosing hybrid event parameters such as timing, state, input, and higher order "shape parameters" to improve the stability of a periodic trajectory. The key insight is that hybrid event shaping introduces a generalizable method to stabilize open-loop hybrid systems.

The saltation matrix is a function of higher order shape parameters h that do not influence the dynamics of the system. These parameters arise from the derivatives of the guards and reset maps, but are not present in the guard, reset map, or vector field definitions themselves. Therefore, shape parameters have absolutely no effect on the nominal trajectory and can be chosen completely freely.

One example of a shape parameter is the angular velocity of a massless leg of a spring-loaded inverted pendulum (SLIP). Since a massless leg does not induce any torque in the air or forces at touchdown, only the position of the leg at touchdown affects the trajectory of the body. However, leg velocity appears in the saltation matrix and has a significant effect on orbital stability [3]. Hybrid event shaping is able to generate optimal retraction rates for stable SLIP hopping and the similar paddle juggling phenomenon [1].

Shape parameters can also be induced with discrete changes in control input. These "virtual hybrid events‘" introduce their own saltation matrices to be shaped.

IV. Example and Results

Consider a fully-actuated compass walker [8] with knees, Fig. 2. This biped model consists of two legs connected by an actuated hip joint. Each leg is separated into two sections, the upper leg (thigh) and lower leg (shank), which are connected by an actuated knee joint that has a hard stop when the thigh and shank are aligned. The ankles are also actuated. The dynamics of this model are described in [1,8].

A direct collocation trajectory optimization method was used with the cost consisting of the stability measure and a regularization on the input. Dynamics and periodicity constraints were included along with a ground penetration constraint.

In this experiment, three trajectories were compared to examine how HES can generate stable trajectories and the effect that virtual hybrid events have in further improving stability. A trajectory without HES was produced as a control, with just an input regularization term in the cost. This minimum energy (ME) trajectory is comparable to conventional robot locomotion trajectories. Two HES trajectories were generated, one with virtual hybrid events (HES w/ VHE) and one without (HES w/o VHE).

The stability properties of the generated trajectories were tested through simulation. 50 trials of each trajectory were initialized with random perturbations. Over a sequence of 10 steps, the state error at each step was tracked for each trial. Fig. 3 shows that after 10 steps, the HES trajectories have nearly converged back to the nominal trajectory whereas the ME trajectories diverge quickly. The HES w/ VHE trajectory converges to a smaller error after 10 steps compared to the HES w/o VHE trajectory.

V. Conclusion

While the idea of hybrid event control is not novel, hybrid event shaping provides a generalized method to analyze the stability of hybrid orbits and select hybrid event parameters to optimize stability. HES unifies results of previous simple hybrid event controllers while also being compatible with trajectory optimization techniques to produce stable trajectories for complex systems.

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