Robust Pivoting Manipulation Using Bilevel Contact-Implicit Optimization

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***Abstract*—In this draft, we present the formulation of bilevel optimization for performing robust pivoting manipulation. We derive analytical expressions for stability margin provided by friction during pivoting manipulation. This margin is then used in the bilevel trajectory optimization to design an open-loop controller that maximizes this stability margin to provide robustness against uncertainty in physical properties of the object. The proposed method hass been tested on several pivoting manipulation scenarios.**

I. Introduction

Contacts are central to most manipulation tasks as they provide additional dexterity to robots to interact with their environment. Designing robust controllers for frictional interaction with objects with uncertain physical properties is challenging as the mechanical stability of the object depends on these physical properties. Inspired by this problem, we consider the task of pivoting manipulation in this paper. In particular, we consider the problem of re-orienting parts with uncertain mass and Center of Mass (CoM) location using pivoting. We are interested in ensuring mechanical stability via friction to compensate for uncertainty in the physical properties of the objects.

We study pivoting manipulation where the object being manipulated has to maintain slipping contact with two external surfaces. A robot can use this manipulation to reorient parts on a planar surface to allow grasping or assist in assembly by manipulating objects to a desired pose (see Fig. 1). Note that this manipulation is challenging as it requires controlled slipping (as opposed to sticking contact [2, 1]), and thus it is imperative to consider robustness of the control trajectories. To ensure mechanical stability of the two-point pivoting in the presence of uncertainty, we derive a sufficient condition for stability which allows us to compute a margin of stability. This margin is then used in a bilevel optimization routine to design an optimal control trajectory while maximizing this margin.

II. Frictional Stability Margin

We briefly provide some physical intuition about frictional stability (see Fig. 2). First, suppose that uncertainty exists in mass of a body. In the case when the actual mass is lower than estimated, the friction force at point A would increase while the friction force at point B would decrease, compared to the nominal case. In contrast, suppose if the actual mass of the body is heavier than that of what we estimate, then the body can tumble along point B in the clockwise direction. In this case, we can imagine that the friction force at point A would

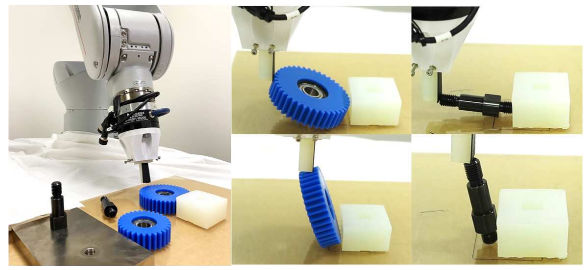

Fig. 1: We consider the problem of reorienting parts for assembly using pivoting manipulation primitive. Such reorientation could possibly be required when the parts being assembled are too big to grasp in the initial pose (such as the gears) or the parts to be inserted during assembly are not in the desired pose (such as the pegs). The figure shows some instances during the implementation of our controller to reorient a gear and a peg.

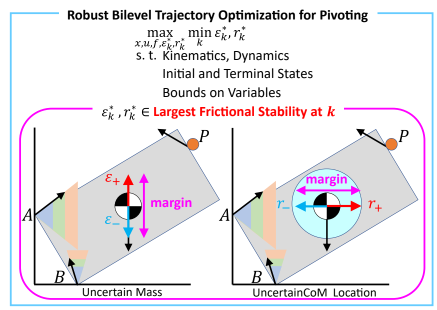

Fig. 2: Conceptual schematic of our proposed frictional stability and robust trajectory optimization for pivoting. Due to slipping contact, friction forces at points A, B lie on the edge of friction cone. Given the nominal trajectory of state and control inputs, friction forces can account for uncertain physical parameters to satisfy static equilibrium. We define the range of uncertainty in gravitational forces and moments that can be compensated by contacts as frictional stability.

decrease while the friction force at point B would increase. However, as long as the friction forces are non-zero, the object can stay in contact with the external environment. Similar arguments could be made for uncertainty in CoM location. The key point to note that the friction forces can re-distribute at the two contact locations and thus provide a margin of stability to compensate for uncertain gravitational forces and moments. We call this margin as *frictional stability*. See [3] for more details.

III. Robust Bilevel Contact-Implicit Optimization

We briefly present our open-loop controller formulation where we incorporate frictional stability in optimization to obtain robustness under uncertaity of mass and CoM location. An important point to note is that the optimization problem

TABLE I: Obtained worst stability margins over the time horizons from optimization for the peg. Note that the stability margin for the solution of the benchmark optimization is analytically calculated.

	ϵ_+^*, ϵ_-^* [N]	r_+^*, r_-^* [mm]
Benchmark optimization	0.035, 0.018	31, 0
Ours (1) with mass uncertainty	0.050, 0.021	N/A
Ours (1) with CoM uncertainty	N/A	38, 0

would be ill-posed if we naively consider uncertainty of mass and CoM location in static equilibrium since there is no u to satisfy all uncertainty realization in equality constraints. Thus, our strategy is that we plan to find an optimal *nominal* trajectory that can ensure external contacts under mass or CoM location uncertainty. In other words, we aim at maximizing the worst-case stability margin over the trajectory given the maximal frictional stability at each time-step k. We formulate a bilevel optimization problem which consists of two lower-level optimization problems as follows:

$$\max_{x,u,f,\epsilon_+^*,\epsilon_-^*} (\min_k \epsilon_{k,+}^* - \max_k -\epsilon_{k,-}^*) \quad \text{(1a)}$$

$$\text{s. t.} \quad \text{kinematics, hybrid dynamics, variable bounds,} \quad \text{(1b)}$$

$$\epsilon_{k,+}^* \in \arg\max_{\epsilon_{k,+}} \{\epsilon_{k,+} : A_k \epsilon_{k,+} \leq b_k, \epsilon_{k,+} \geq 0\}, \quad \text{(1c)}$$

$$\epsilon_{k,-}^* \in \arg\max_{\epsilon_{k,-}} \{\epsilon_{k,-} : -A_k \epsilon_{k,-} \leq b_k, \epsilon_{k,-} \geq 0\} \quad \text{(1d)}$$

where $\epsilon_{k,+}^*, \epsilon_{k,-}^*$ are the largest uncertainty of mass in the positive and negative direction, respectively, at k given x, u, f, which results in non-zero contact forces. x, u, f are states (i.e., box orientation and position of point P in Fig. 2), control input (i.e., forces at point P), and external contact forces from point A and B, respectively. A_k, b_k represent inequality constraints associated with frictional stability margin. (1a) is the upper-level objective function, which maximizes the smallest stability margin over time-horizons by subtracting the stability margin along the positive direction from that along the negative direction.

The advantage of (1) is that since the lower-level optimization problem are formulated as two linear programming problems, we can efficiently solve the entire bilevel optimization problem using the KKT condition. Using the KKT condition and epigraph tricks, (1) becomes a single-level large-scale nonlinear programming problem with complementarity constraints. See [3] for more details.

IV. Experimental Results

We first show some numerical results discussing how much robustness our proposed controller provide over the baseline controller as shown in Table I and Fig. 3. Fig. 3 shows the frictional stability for the peg obtained during the trajectories from the proposed method and the baseline method.

Table I summarizes the computed stability margin from Fig. 3. Our bilevel optimizer finds trajectories that have larger stability margins for both uncertain mass and CoM location. The trajectory of stability margin obtained from bilevel optimization considering mass uncertainty is illustrated in Fig. 3.

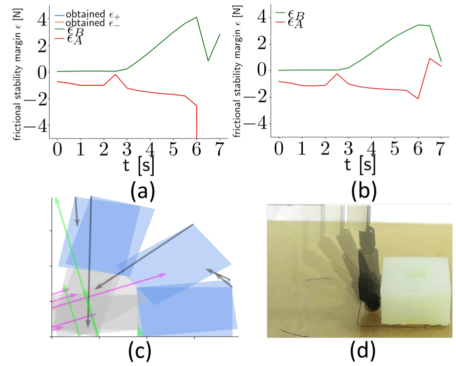

Fig. 3: (a), (b): Trajectory of frictional stability margin of the peg based on uncertain mass obtained from our proposed bilevel optimization, baseline optimization, respectively. (c): Snapshots of pivoting motion for the peg, obtained from our proposed bilevel optimization considering uncertain mass. (d): Snapshots of hardware experiments for the peg.

TABLE II: Number of successful pivoting attempts of gear 1 over 10 trials for the two different methods. To evaluate robustness for objects with unknown mass, we solve the optimization with mass different from the known object and implement the obtained trajectory on the object with known mass. Note that the actual mass of the gear is 140 g.

	Bilevel Optimization	Benchmark Optimization
$m = 100$ g	10 / 10	0 / 10
$m = 110$ g	10 / 10	0 / 10
$m = 140$ g	10 / 10	0 / 10
$m = 170$ g	10 / 10	0 / 10

We also implement our controller using a real 6 DoF manipulator to demonstrate the efficacy of our proposed method for various gears and pegs. To evaluate robustness for objects with unknown mass, we solve the optimization with mass different from the true mass of the object and implement the obtained trajectory on the object.

Table II shows the success rate of pivoting for the hardware experiments. We observe that our proposed bilevel optimization is able to achieve 100 % success rates for all 4 mass values while benchmark optimization cannot realize stable pivoting.

References

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